



SURFACE VEHICLE RECOMMENDED PRACTICE

J2789™

APR2023

Issued 2010-08
Revised 2018-09
Stabilized 2023-04

Superseding J2789 SEP2018

Inertia Calculation for Single-Ended Inertia-Dynamometer Testing

RATIONALE

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1. SCOPE

This procedure provides methods to determine the appropriate inertia values for all passenger cars and light trucks up to 4540 kg of GVWR. For the same vehicle application and axle (front or rear), different tests sections or brake applications may use different inertia values to reflect the duty-cycle and loading conditions indicated on the specific test.

1.1 Purpose

The purpose of this procedure is to provide a common methodology to calculate test inertia (wheel load and tire rolling radius). The use of common inertia values allows the comparison of test results from different inertia-dynamometers or different testing facilities. This makes the overall testing activities more cost-effective and repeatable.

2. REFERENCES

2.1 Related Publications

The following publications are provided for information purposes only and are not a required part of this SAE Technical Report.

2.1.1 ISO Publications

Copies of these documents are available online at <http://webstore.ansi.org/>.

ISO 611 Road Vehicles - Braking of Automotive Vehicles and Their Trailers - Vocabulary

3. DEFINITIONS

3.1 ANTILOCK BRAKING SYSTEMS - ABS

Closed-loop control device which prevents wheel lock when braking and, as a result, retains the vehicle's steering ability and stability.

3.2 ELECTRONIC BRAKE DISTRIBUTION - EBD

Closed-loop technology that automatically varies the amount of pressure applied to the rear brakes to maximize tire-to-road adhesion utilization while maintaining the vehicle stability. This condition typically happens at or near the wheel lock-up condition.

3.3 GROSS VEHICLE WEIGHT RATING - GVWR

Maximum vehicle weight indicated by the manufacturer. [kg]

3.4 LIGHTLY LOADED VEHICLE WEIGHT - LLVW

Unloaded vehicle weight plus 180 kg including driver and test instrumentation. [kg]

3.5 TIRE DYNAMIC ROLLING RADIUS - RR

Tire radius that equates to the Revolutions Per Mile (RPM) published by the tire manufacturer for the specific tire size. If unknown, the rolling radius can be calculated from the RPM value using Equation 1. Use the tire dynamic rolling radius to calculate test inertia and the dynamometer rotational speed in revolutions per minute (r/min) for a given linear vehicle speed.

$$RR = \frac{1\,609.344}{2 \cdot \pi \cdot RPM} \quad (\text{Eq. 1})$$

where:

RR = Tire dynamic rolling radius [m]

RPM = Tire manufacturer specification for revolutions per mile. Typically shown for the tire size on the manufacturer's website.

4. TEST INERTIA CALCULATION METHODS

There are three methods available to calculate the test inertia depending upon the amount of information available, the vehicle type, and the deceleration level for the specific test section or brake application.

The inertia values are required to determine the amount of energy imposed on the brake during testing. Also, the inertia value is required to determine the torque level for a given deceleration value or set-point.

The minimum information required to run an inertia-dynamometer test is:

- Vehicle Description: make or manufacturer, model year, brand or platform, trim level
- Gross Vehicle Weight Rating
- Lightly Loaded Vehicle Weight
- Tire Size Description
- Brake Proportioning Type (fixed or electronic)

Use the flowchart in Figure 1 to determine the appropriate method as a function of availability of vehicle information, type of front-to-rear brake proportioning, and target deceleration. Follow the steps on the flowchart, answer the questions and then proceed to the corresponding section using the information available for the specific vehicle application.

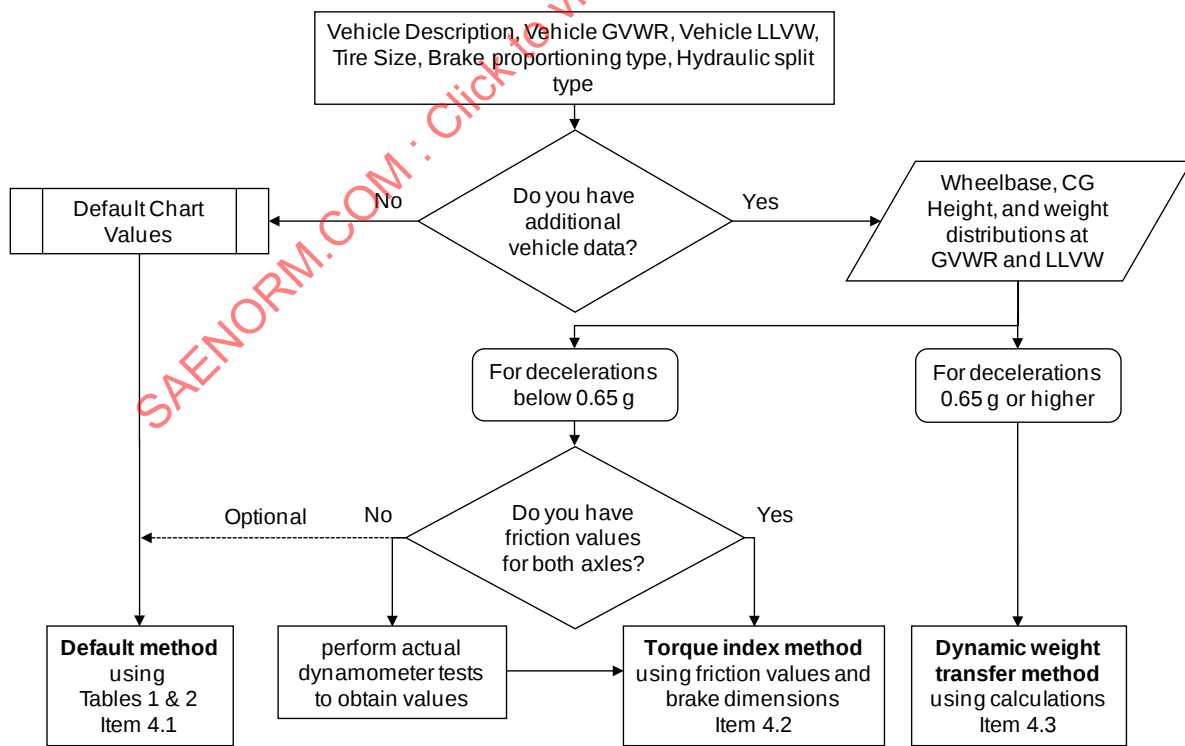


Figure 1 - Workflow to select the inertia calculation method

There are three main methods to determine the corresponding inertia level for the test as a function of the brake system configuration and the level of information available at the time of testing:

- Default method for front-to-rear inertia split when there is no detailed vehicle information available. See 4.1.
- Torque index method using nominal brake torque output for the front and rear brake as a function of known friction values and the corresponding brake dimensions. This method applies to fixed brake proportioning or for ABS/EBD system at vehicle decelerations below 0.65 g. See 4.2.
- Dynamic weight transfer method for all vehicle decelerations above 0.65 g when there is detailed vehicle information available. For vehicles with fixed proportioning, use the LLVW values for all weight conditions. See 4.3.

The test inertias follow the front-to-rear braking force distribution. Equations 2 and 3 provide the calculation for the front and rear inertias.

$$I_{front} = \frac{1}{2} \cdot X \cdot W \cdot RR^2 \quad (\text{Eq. 2})$$

$$I_{rear} = \frac{1}{2} \cdot Y \cdot W \cdot RR^2 \quad (\text{Eq. 3})$$

where:

I_{front} = test inertia for front brake [kg·m²]

I_{rear} = test inertia for rear brake [kg·m²]

X and Y = Percentage of brake torque provided by the front or rear axle respectively. See 4.1 and 4.2 for different methods to obtain them based on default values, nominal torque calculations, or actual dynamometer testing.

W = vehicle test weight as specified by the requestor: GVWR or LLVW [kg]

RR = tire dynamic rolling radius per item Equation 1 [m]

NOTE: Verify with the test requestor if there is a loss factor required to adjust the calculated inertia from Equations 2 and 3 to account for engine braking, transmission losses, or aerodynamic drag.

For brake applications under hydraulic circuit failure use one of the two following calculation depending upon the type of brake system split:

- For diagonal hydraulic circuit split use twice the inertia calculated for the corner under testing
- For front-to-rear hydraulic circuit split use half the inertia for the total vehicle (LLVW or GVWR)

4.1 Default Method

Use the values provided in Tables 1 or 2 for fixed or electronic brake proportioning respectively.

These values are not appropriate for final brake system sizing, performance prediction using dynamometer test results or for investigating brake performance for a specific vehicle configuration when the brake system, weight distribution, and location of the center of gravity are known.

Since the late 1980s, all full-size pickup trucks manufactured in the U.S. have at least a rear wheel antilock brake system. Therefore, use the electronic proportioning work split table and calculations for these vehicles.

The sum of the front and rear inertias adds to more than 100% to take into account the variation in vehicles within the same category.

The default table uses the following boundary conditions for fixed proportioning (including proportioning valves):

- The brake work distribution is based under the assumption that the proportioning valve setting ensures front-bias at LLVW.
- Since the proportioning valve has fixed settings, the pressure distribution at GVWR will be the same as at LLVW.
- At low deceleration, the line pressure is typically below or close to the split point of the proportioning valve. Hence, the valve will have little or no effect; and the vehicle will be less front-biased than above the split point.

The default table uses the following boundary conditions for Electronic Brake Distribution systems:

- Vehicles with EBD usually have a g-critical at LLVW between 0.6 and 0.8 g. Compared to vehicles that are not equipped with EBD, there will be a higher percentage of braking work done by the rear brakes at GVWR and at low decelerations at the LLVW condition.
- At low decelerations, the EBD would probably not be active so the LLVW and GVWR brake force distributions would be the same.

Table 1 - Inertia split for fully-operational systems with fixed brake proportioning

Percent of brake force done by each axle (X and Y values)								
Vehicle type	Fixed Proportioning							
	Low deceleration < 0.65 g				High deceleration > 0.65 g			
	GVWR		LLVW		GVWR		LLVW	
	Front X	Rear Y	Front X	Rear Y	Front X	Rear Y	Front X	Rear Y
Passenger car - FWD	78	28	78	28	80	25	80	25
Passenger car - RWD	78	28	78	28	75	30	75	30
Minivan and crossover	78	28	78	28	75	30	75	30
Pick-up trucks	68	38	63	45	80	25	80	25
SUV-RWD	73	33	73	33	75	30	75	30

Table 2 - Inertia split for fully-operational systems with electronic brake distribution

Percent of brake force done by each axle (X and Y values)								
Vehicle type	Electronic brake distribution							
	Low deceleration < 0.65 g				High deceleration > 0.65 g			
	GVWR		LLVW		GVWR		LLVW	
	Front X	Rear Y	Front X	Rear Y	Front X	Rear Y	Front X	Rear Y
Passenger car - FWD	70	35	70	35	70	35	80	25
Passenger car - RWD	68	38	68	38	68	38	75	30
Minivan and crossover	70	35	70	35	70	35	80	30
Pick-up trucks	55	50	60	45	60	45	80	25
SUV-RWD/AWD	65	40	65	40	65	40	75	30

4.2 Torque Index Method

If there is information available from the requestor regarding the friction or effectiveness values for both axles, in addition to the brake sizes, calculate the values for X and Y required for Equations 2 and 3 using the values obtained from the torque index BD and Equations 6 and 7. Use Equation 4 to calculate the torque index for all-disc brake system configurations. Use Equation 5 to calculate the torque index for disc-drum brake system configurations:

$$BD_{disc-disc} = \frac{A_{p-rear} \cdot r_{eff-rear} \cdot \mu_{rear}}{A_{p-front} \cdot r_{eff-front} \cdot \mu_{front}} \quad (\text{Eq. 4})$$

$$BD_{disc-drum} = \frac{A_{p-rear} \cdot r_{eff-rear} \cdot C^*}{2 \cdot A_{p-front} \cdot r_{eff-front} \cdot \mu_{front}} \quad (\text{Eq. 5})$$

$$X = \frac{1}{1+BD} \quad (\text{Eq. 6})$$

$$Y = \frac{BD}{1+BD} \quad (\text{Eq. 7})$$

where:

BD = torque index as a ratio of rear to front torque output at the same input pressure

A_p = total piston area acting on one side of the caliper for disc brakes; total wheel cylinder area for drum brakes [mm²]

r_{eff} = radial distance from centerline of the piston to the axis of rotation for disc brakes; internal drum diameter divided by 2 for drum brakes, unless other dimensions are provided by the requestor [mm]

μ = apparent friction for disc brakes [unitless]

C^* = effectiveness for drum brakes [unitless]

If there is no information available from the requestor regarding the friction or effectiveness values for both axles, conduct partial inertia-dynamometer testing per SAE J2784 to calculate the values for X and Y as follows:

- Perform separate inertia dynamometer testing on the front and the rear brakes
- Obtain apparent friction or effectiveness (C^* values) from the test results
- Calculate the corresponding value for BD using Equation 4 or 5 for disc-disc or disc-drum configuration respectively
- Calculate the corresponding values for X and Y using Equations 6 and 7
- Calculate the values for the front and rear inertias using Equations 2 and 3 respectively

Detailed steps are as follows:

- Determine the appropriate burnish section from the SAE J2784 Recommended Practice as a function of the Gross Vehicle Weight Rating: for vehicles 3500 kg or below GVWR use Table 2; for vehicles above 3500 kg GVWR use Table 3.
- Setup the test inertia for the front brake. Unless indicated by the requestor, setup the test inertia per the values provided in Table 1 or 2 corresponding to low deceleration.

- c. Perform one complete burnish sequence on the front brake and compute the average apparent friction value from the last 10 burnish stops.
- d. Change over brake fixture and test inertia to the rear brake. Unless indicated by the requestor, setup the test inertia per the values provided in Table 1 or 2 corresponding to low deceleration.
- e. Perform one complete burnish sequence on the rear brake and compute the average apparent friction or effectiveness (C^* values) the last 10 burnish stops.

Using the friction values obtained during the last 10 burnish stops from both dynamometer tests, setup the inertia for the actual test using the calculations indicated in this item.

4.3 Dynamic Weight Transfer Method

For deceleration levels above 0.65 g, the test inertias need to reflect the dynamic weight transfer that occurs as a function of the braking deceleration, the location of the vehicle's center of gravity, and the weight distribution.

If there is information available from the requestor regarding the vehicle mass distribution, center of gravity locations, and vehicle wheel base, use the following set of equations to determine the dynamic wheel load at each axle. This dynamic wheel load in conjunction with the tire dynamic rolling radius provides the values for the test inertia at the specific test section. Certain test procedures indicate specific vehicle configurations (operational or failed), weight conditions (GVWR or LLVW) depending upon the vehicle condition that the particular section recreates or represents. The calculation uses different deceleration levels for the front and rear axles (0.9 g and 0.65 g, respectively) to consider what are critical conditions when assessing that corner in particular.

For vehicles with fixed brake proportioning, use the front-to-rear split calculation at LLVW for all test weight conditions.

$$I = W_{dyn} \cdot RR^2 \quad (\text{Eq. 8})$$

$$W_{f-dyn} = \frac{1}{2} \left[1 - \frac{W_{r-static}}{W} + \frac{h \cdot a}{L} \right] \cdot W \quad (\text{Eq. 9})$$

$$W_{r-dyn} = \frac{1}{2} \left[\frac{W_{r-static}}{W} - \frac{h \cdot a}{L} \right] \cdot W \quad (\text{Eq. 10})$$

where:

I = test inertia; front or rear brake [$\text{kg} \cdot \text{m}^2$]

W_{dyn} = test dynamic wheel load, W_{f-dyn} or W_{r-dyn} [kg]

W_{f-dyn} = dynamic wheel load on front brake [kg]

W_{r-dyn} = dynamic wheel load on rear brake [kg]

$W_{r-static}$ = static axle load at test weight (GVWR or LLVW) [kg]

W = total vehicle test weight (GVWR or LLVW) [kg]

h = center of gravity height measured from the ground [m]

a = reference deceleration level. Unless indicated by the requestor, use 0.9 for front brakes and 0.65 for rear brakes [g].

L = vehicle wheel base [m]

NOTE: Verify with the test requestor if there is a loss factor required to adjust the calculated inertia from Equation 8 to account for engine braking, transmission losses, or aerodynamic drag.

5. INERTIA CORRECTION FOR VEHICLES WITH REGENERATIVE BRAKING

Different from the legacy Internal Combustion Engines (ICE) where all the braking forces are provided by the foundation brakes, newer non-ICE powertrain technologies use some level of braking energy regeneration (recuperation) in addition to the frictional brakes mounted at the wheels. Some new powertrain and prime mover systems are:

- a. Micro Hybrid using the electric motor only for limited start/stop functions.
- b. Mild Hybrids using a combustion engine (gasoline or diesel) supported by low-power electric motor(s), but cannot run entirely on electric power.
- c. Full Hybrids (HEV) using mainly a combustion engine along with electric motor(s) that can power the vehicle for short distances.
- d. Plug-in Hybrids (PHEV), similar to full-hybrids with electric motor(s) capable of powering the vehicle for longer distances. In addition, the high-voltage batteries can be recharged by connecting them to the power grid (external charging at home or at public charging stations).
- e. Electric Vehicles (BEV) powered only by electric motor(s). The high-voltage battery system is recharged mainly by connecting them to the power grid.
- f. Electric Vehicles with Range Extenders (EREV, REEV, RXBEV, or BEVx) with electric motor(s) and a small combustion engine to charge the battery during vehicle operation.
- g. Electric Vehicles with Fuel Cell (FCBEV or FCEV) with electric motor(s) powered by a hydrogen fuel cell.

In addition, there are at least three main powertrain configurations to connect the prime movers (internal combustion engine or electric motors) to the wheels for acceleration and braking:

- a. Parallel hybrid on which either or both power source (internal combustion engine or electric motors) can provide propulsion to the vehicle. This type of powertrain relies more on regenerative braking, and tends to have smaller battery packs, relying on regenerative braking to keep it recharged. When power demands are low, parallel hybrids use the motor as a generator for supplemental recharging, much like an alternator in conventional cars. An alternative to the parallel hybrid powertrain is the Through the Road (TTR) with a conventional powertrain (ICE) to drive one axle, and electric motor(s) to drive the other axle.
- b. Series hybrid where the electric motor(s) are coupled directly to the axle, or the wheel, and are the simplest hybrid configuration. In a series hybrid, the electric motor is the only source of power to the wheels. The motor receives electric power from either the battery pack or from a generator run by an ICE. A computer algorithm determines how much of the power comes from the battery or the engine/generator. Both the engine/generator and the use of regenerative braking recharge the battery pack.
- c. Power-split hybrid or series-parallel hybrid are parallel hybrids that incorporate power-split devices. This configuration allows for power paths from the ICE to the wheels that can be either mechanical or electrical. The main principle is to decouple the power supplied by the primary source from the power demanded by the driver.

There are two main methods to reflect the interaction of the regenerative and the foundation brake systems.

5.1 Real Time (Speed Domain) Simulation of Regenerative (Regen) Braking

This is when the project has access to (a) full specification of the regenerative and vehicle battery systems, and (b) an inertia dynamometer which has the ability (inertia simulation and control algorithms) to change the inertia reflected on the foundation brake in real time. In this testing scenario, the dynamometer can reproduce braking profiles to reflect the interaction between the battery system, the vehicle speed and deceleration behavior, and the required foundation brake torque output. During regenerative (regen) braking the electric motor (acting as a generator) works in conjunction with the friction (foundation) brake to provide constant torque (deceleration) braking. For this method, review with the test requestor and the test facility before the project: (a) the extent of real time regen simulation feasible and required, (b) the type of brake events and test sections on which to use regen simulation, and (c) the specific algorithms (including validation prior to actual testing) to implement the regen simulation. Depending upon the battery pack size (power, voltage, and allowed charge levels), the powertrain configuration (parallel, series, or series-parallel), the control algorithms, and the vehicle demands for braking (hill grade, vehicle size and weight, and total deceleration level), there are at least three distinct regions for the torque blending. See Figure 2.

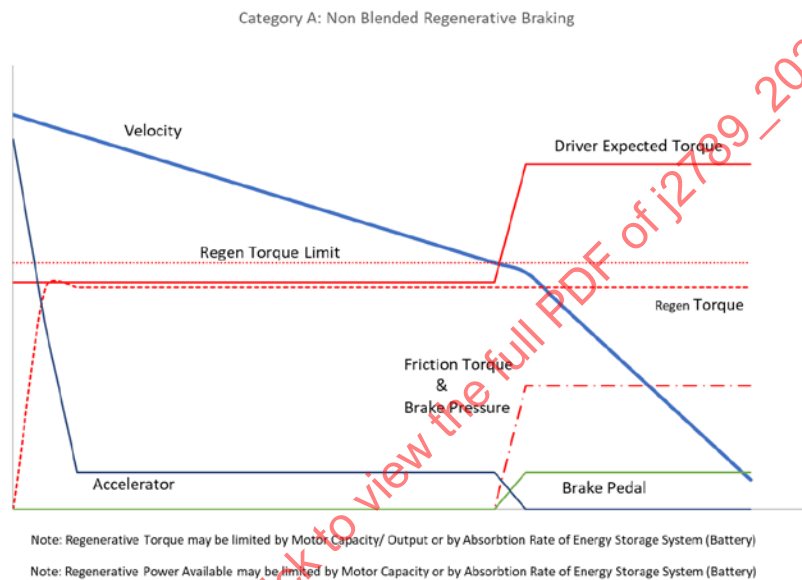


Figure 2A - Category a for non-blended regenerative braking